

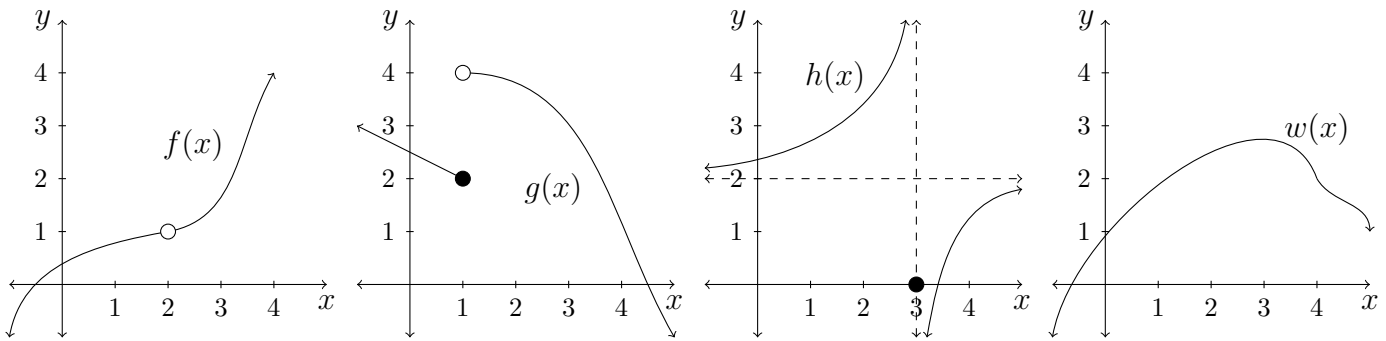
Goals:

- Define limits of functions (one-sided and two-sided).
- Compute limits of functions graphically and numerically.

Intuition:

The limit of a function asks “what value is this function getting near to?” This is not always the same as the value of the function.

Graphically:



$$\lim_{x \rightarrow 2} f(x)$$

$$f(2)$$

$$\lim_{x \rightarrow 1^+} g(x)$$

$$\lim_{x \rightarrow 1^-} g(x)$$

$$\lim_{x \rightarrow 1} g(x)$$

$$g(1)$$

$$\lim_{x \rightarrow 3^-} h(x)$$

$$\lim_{x \rightarrow 3^+} h(x)$$

$$\lim_{x \rightarrow 3} h(x)$$

$$h(3)$$

$$\lim_{x \rightarrow \infty} h(x)$$

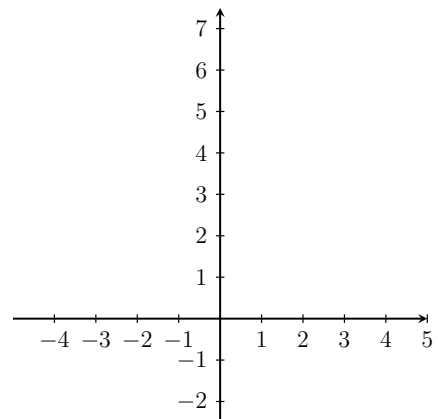
$$\lim_{x \rightarrow 4} w(x)$$

$$w(4)$$

Numerically:

x	$\frac{e^x - 1}{x}$
0	
0.01	
-0.01	
0.001	

Guess $\lim_{x \rightarrow 0} \frac{e^x - 1}{x}$:



Some Definitions:

- **LIMIT** If we can make the values of $f(x)$ arbitrarily close to L by taking x to be sufficiently close to a (on either side of a) but not equal to a , we say “the limit of $f(x)$, as x approaches a , equals L ” and we write,

- **RIGHT-HAND LIMIT** We say “the right-hand limit of $f(x)$, as x approaches a [or the limit of $f(x)$ as x approaches a from the right], equals L ” and we write,

if we can make the values of $f(x)$ arbitrarily close to L by taking x to be sufficiently close but not equal to a AND _____

- **LEFT-HAND LIMIT** We say “the left-hand limit of $f(x)$, as x approaches a [or the limit of $f(x)$ as x approaches a from the left], equals L ” and we write,

if we can make the values of $f(x)$ arbitrarily close to L by taking x to be sufficiently close but not equal to a AND _____

FACT: For a function $f(x)$,

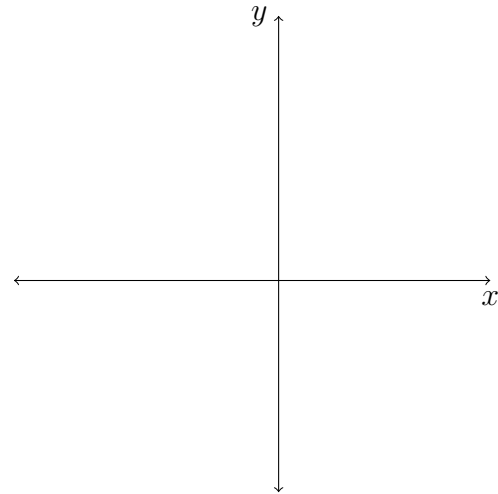
$\lim_{x \rightarrow a} f(x) = L$ if and only if _____ and _____ .

Consider the function

$$f(x) = \frac{x + 2}{x^2 - 5x - 14}$$

Use at least five values of x to approximate $\lim_{x \rightarrow -2} f(x)$ and sketch the graph (including the scale).

x	$\frac{x + 2}{x^2 - 5x - 14}$



Sketch a function $f(x)$ satisfying the following:

- $\lim_{x \rightarrow -\infty} f(x) = 0$
- $\lim_{x \rightarrow \infty} f(x) = \infty$
- $\lim_{x \rightarrow -1^-} f(x) = -2$
- $\lim_{x \rightarrow -1^+} f(x) = 2$
- $f(-1) = 0$
- $\lim_{x \rightarrow 1^+} f(x) = 2$
- $\lim_{x \rightarrow 1^-} f(x) = -1$

